5

Unit

Fundamentals of Solid State Physics

Enrico Fermi
(1901 – 1954)

Learning Objectives:

After studying this chapter, students should be able to:

- Understand the term Crystal, Basis, Lattice, Unit cell.
- Describe the two and three dimensional lattice.
- Know about types of Cubic lattice, Coordination number, Atomic radius, Packing fraction.
- Describe the crystal structure of NaCl, CsCl, HCP and Diamond.
- Describe the different types of crystal bonding.
- Know about the free electron theory.
- Explain the Classical free electron theory and its limitations.
- Describe the Quantum mechanical free electron model.
- Understand the Fermi level and Fermi energy.
- Derive the Fermi energy and density of state of free particle.
- Describe the effect of temperature on Fermi Dirac distribution.
- Write the success and failure of free electron theory.
- Know the concept of band theory and distinguish the Conductors, Semiconductors and Insulators based on band theory.
- State and explain the Bloch function and theorem.
- Describe the Kronig-Penny model.
- Know about the effective mass of particle.
- Solve the various numerical problems related with Solid State Physics.

splitted into n additional energy levels. Such a large number of splitted energy levels are so closely packed into a range of energy that the single energy level seems to be broadened rather than splitted. The range of energy in which all the splitted levels of an energy level of an atom falls is called energy band.

7. How are solids classified according to the band theory of energy?

Ans: On the basis of the widths of the energy band in solids, the solids are classified into conductor, insulator and semiconductor. In conductors the valence band and conduction band overlap each other. Due to this overlapping a slight potential difference across a conductor can cause the free electrons to constitute electric current. In insulators, the forbidden energy gap is wide, the valence band is full and the conduction band is empty. As a result there is no conduction at normal temperature. In semiconductors, the valence band is almost filled and the conduction band is nearly empty with a small forbidden energy gap (= 1eV) between them. The conductivity of a semiconductor lies between a good conductor and an insulator.

8. Differentiate between conductor and semiconductor.

Ans: The difference between conductor and semiconductor are as given below:

Conductor	Semiconductor
1. The free electrons are responsible for conduction of electricity. 2. It has very large conductivity. 3. It has positive temperature coefficient of resistance. 4. There is no forbidden energy gap between valence band and conduction band.	1. Both electrons and holes are responsible for conduction of electricity. 2. It's conductivity lies between conductor and insulator. 3. It has negative temperature coefficient of resistance. 4. A small forbidden energy band exists between conduction band and valence band.

Worked Out Examples

1. Consider a copper wire of cross-section area 1 mm² carrying a current 1A. What is the drift velocity of the electron? The density and molecular weight of Cu are 9 gm/cm³ and 64 g/mole respectively. ***[TU Microsyllabus 2074, W; 23.1]***

Solution:

Area of Copper wire (A) = 1 mm² = 10^{-6} m²

Current (I) = 1A

Density of copper (ρ) = 9 gm/cm³

Molecular weight = 64 gm/mol.

Drift velocity of electron (V_d) = ?

We know that,

$$V_d = \frac{J}{Nq}$$

The current density $J = \frac{I}{A} = \frac{1}{10^{-6}} = 10^6$ Amp/m².

For copper (monovalent), the number of free electrons per unit volume N is equal to the number of atoms Per unit volume N_{atoms}. So that,

N_{atoms} = Number of moles/cm³ × number of atoms/mole

Where, the number of atoms per mole is given by Avogadro's number $N_A = 6.02 \times 10^{23}$ atoms/mole. thus,

$$N = N_{atoms} = \frac{9\ \text{gm/cm}^3}{64\ \text{gm/mole}} \times (6.02 \times 10^{23}\ \text{atoms/mole})$$

$$= 8.4 \times 10^{22}\ \text{atoms/cm}^3 = 8.4 \times 10^{28}\ \text{atoms/m}^3$$

$$\therefore\ V_d = \frac{10^6}{8.4 \times 10^{28} \times 1.6 \times 10^{-19}} = 7 \times 10^{-5}\ \text{m/sec.}$$

Thus, required drift velocity (V_d) = 7×10^{-5} m/sec.

2. Consider a current carrying copper wire, the number of free electrons in copper is 8.4×10^{28} electron/m³. (a) Calculate the Fermi energy for Cu (b) At what temperature, T_f will the average thermal energy K_BT_F of a gas be equal to that energy?

[TU Microsyllabus 2074, W; 23.2]

Solution:

Number of free electron in copper (N) = 8.4×10^{28} electron/m³

Mass of electron (m) = 9.1×10^{-31} kg

a. We know that, the expression of Fermi energy at T = 0K is

$$E_f(0) = \frac{h^2}{2m}(3N\pi^2)^{2/3}$$

$$E_f(0) = \frac{1.05 \times 10^{-34}}{2 \times 9.1 \times 10^{-31}}(3 \times 8.4 \times 10^{28} \times \pi^2)^{2/3}$$

$$E_f(0) = 11.1 \times 10^{-19}\ \text{J} = 6.95\ \text{eV}$$

b. We know that $E_f(0) = K_BT_f$. Where, K_B = Boltzmann constant = 1.38×10^{-23} J/k and T_f = Fermi Temperature. Thus,

$$T_f = \frac{E_f(0)}{K_B} = \frac{11.1 \times 10^{-19}\ \text{J}}{1.38 \times 10^{-23}\ \text{J/k}} = 80{,}500\ \text{K}$$

Hence, the required Fermi energy and Fermi temperature are 6.95 eV and 80,500 K respectively.

3. The electrical conductivity of Cu at room temperature is $5.9 \times 10^7\ \Omega^{-1}\ m^{-1}$. The Fermi energy for copper is 6.95 eV and carrier density 8.4×10^{28} electrons/m³. Calculate the mean free path of the electrons.

Solution:

Electrical conductivity (σ) = $5.9 \times 10^7\ \Omega^{-1}\ m^{-1}$

Carrier density (N) = 8.4×10^{28} electons/m³

Fermi energy (E_f) = 6.95 eV

Mean free path (l) = ?

Fermi velocity can be obtained by $(E_f) = \frac{1}{2}mv_f^2$

$$v_f = \left(\frac{2E_f}{m}\right)^{\frac{1}{2}} = \left(\frac{2 \times 6.95 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}\right)^{\frac{1}{2}} = 1.56 \times 10^6\ \text{m/sec}$$

Again, we have the relation of electric conductivity

$$\sigma = \frac{Nq^2\tau}{m}$$

$$\text{or,}\quad \tau = \frac{\sigma m}{Nq^2} = \frac{5.9 \times 10^7 \times 9.1 \times 10^{-31}}{8.4 \times 10^{28} \times (1.6 \times 10^{-19})^2}$$

$$\therefore\quad \tau = 2.50 \times 10^{-14}\ \text{sec}$$

We know that, the mean free path

$$l = v_f\tau = 1.56 \times 10^6 \times 2.50 \times 10^{-14}$$

$$\therefore\quad l = 3.90 \times 10^{-8}\ \text{m} = 390\text{Å}$$

Hence, required mean free path of electrons is 390 Å.

4. The electrical conductivity of copper at 300 K is $5.9 \times 10^7\ \Omega^{-1}m^{-1}$ (a) What is the thermal conductivity at room temperature (b) What is the thermal conductivity at 1000 K?

Solution:

The electrical conductivity at temperature 300 K is (σ) = $5.9 \times 10^7\ \Omega^{-1}m^{-1}$

Thermal conductivity at room temperature (K_1) = ?

a. We know that Wiedemann Frantz law, $\frac{K}{\sigma T} \propto L = 2.45 \times 10^{-8}\ W\Omega/deg^2$

$$\frac{K_1}{\sigma T} = \frac{\pi^2}{3}\left(\frac{K_B}{e}\right)^2 = 2.45 \times 10^{-8}\ w\Omega/deg^2 = L\ (\text{say})$$

$$K_1 = \sigma_1 T_1 L = 5.9 \times 10^7 \times 300 \times 2.45 \times 10^{-8} = 407.1\ \text{W/mk}.$$

b. We know that,

$\frac{\sigma_1}{\sigma_2} = \frac{T_2}{T_1}$ Since, $\sigma \propto \frac{1}{T}$

$\frac{\sigma_{300}}{\sigma_{1000}} = \frac{1000}{300}$

or, $\sigma_{1000} = \frac{300 \times \sigma_{300}}{1000}$

∴ $\sigma_{1000} = \frac{300 \times 5.9 \times 10^7}{1000} = 1.77 \times 10^7\ \Omega^{-1}m^{-1}$

Thus, $K_2 = \sigma_2 T_2 L = 1.77 \times 10^7 \times 1000 \times 2.45 \times 10^{-8}$

∴ $K_2 = 41.92$ w/mk

Hence, required thermal conductivity at 300 K and 1000 K are 407.1 W/mK and 41.92 W/mK respectively.

5. **Copper has a face-centered cubic structure with a one-atom basis. The density of copper is 8.96 gm cm^{-3} and its atomic weight is 63.5 g $mole^{-1}$. What is the length of the unit cube of the structure?** ***[TU Microsyllabus 2074 P; 21.1 TU Exam 2074]***

Solution:

Here is given, density of copper (ρ) = 8.96 g cm^{-3}

Number of atom per unit cell (N) = 4 Since, Fcc structure

Avargado number (N_A) = 6.023×10^{23} atoms $mole^{-1}$

Length of the unit cube (a) = ?

Atomic weight of copper (m) = 63.5 g $mole^{-1}$

We know that,

$\frac{N}{V} = \frac{\rho N_A}{m}$

or, $\frac{N}{a^3} = \frac{\rho N_A}{m}$ Since, volume of unit cell = a^3

or, $a^3 = \frac{Nm}{\rho N_A} = \frac{4 \times 63.5}{8.96 \times 6.023 \times 10^{23}}$

or, $a^3 = 4.71 \times 10^{-23}$

or, $a = 3.61 \times 10^{-7}$ cm

∴ a = 3.61 Å Since, 10^{-8} cm = 1 Å

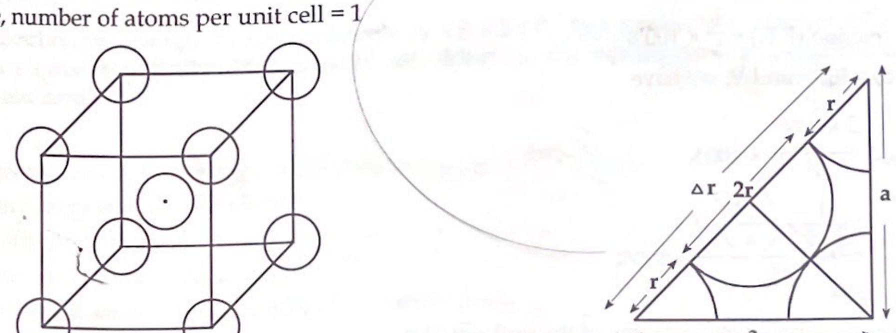
Hence, the required length of the unit cubic of the structure is 3.61 Å.

6. **Assuming that atoms in a crystal structure are arranged in close-packed spheres, what is the ratio of the volume of the atoms to the volume available for the simple cubic structure? Assume a one-atom basis.** ***[TU Microsyllabus 2074, P; 22.3]***

Solution:

Each corner atom in a cubic unit cell is shared by a total number of eight unit cells so that each corner atom contributes only $\frac{1}{8}$ of its effective part to a unit cell. Since, there are in all 8 corner atoms their total contribution is equal to $\frac{8}{8} = 1$

Therefore, number of atoms per unit cell = 1



[Fig. 66: Representation of simple cubic structure of unit cell]

From figure 66,

$$a = 2r$$

or, $r = \frac{a}{2}$

Therefore, Volume occupied by the atom in the unit cell $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

Volume of unit cell $= a^3$

Thus, Packing fraction $= \dfrac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{a^3} = \frac{\pi}{6} = 0.52 = 52\%$

Therefore, 52% space of unit cell is occupied by the atoms. Here the atoms are loosely packed. Only Polonium at a certain temperature is known to exhibit such a structure. For example, KCl which has alternate ions of K and Cl also behaves like a simple lattice as regards scattering of X-rays because the two ions are almost identical.

7. **Assuming that atoms in a crystal structure are arranged in close-packet spheres, what is the ratio of the volume of the atoms to the volume available for the body-centered cubic structure? Assume a one-atom basis.** *[TU Microsyllabus 2074, P; 22.4]*

Solution:

For a body cantered unit cell, the atomic radius can be calculated from figure 67 as follows. From figure, AH = 4r and DH = a

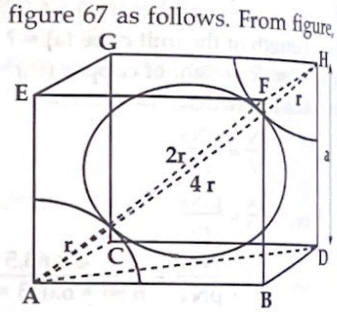
From the triangle AHD,

$$AH^2 = AD^2 + DH^2 \quad \text{...(1)}$$

From the triangle ABD

$$AD^2 = AB^2 + BD^2$$

$$AD^2 = a^2 + a^2 \quad \text{... (2)}$$



[Fig. 67: Representation of body centred cubic structure]

Substituting equation (2) in (1), we get

$$AH^2 = 2a^2 + a^2$$

or, $AH^2 = 3a^2$

or, $(4r)^2 = 3a^2$

or, $16r^2 = 3a^2$

or, $r^2 = \frac{3a^2}{16}$

or, $r = \frac{\sqrt{3}}{4}a$

Packing Fraction $= \dfrac{\text{Number of atoms present per unit cell} \times \text{Volume of atom}}{\text{Volume of the Unit Cell}}$

Number of atoms per unit cell = 2

Volume of 2 atoms, $v = 2 \times \frac{4}{3}\pi r^3$

Side of the unit cell, $a = \frac{4r}{\sqrt{3}}$ Since, atomic radius, $r = \frac{\sqrt{3}}{4}a$

Volume of the unit cell, $V = a^3$

Packing fraction (P.F.) $= \frac{v}{V} \times 100\%$

Substituting for v and V, we have

$$\therefore \quad \text{P.F.} = \frac{2 \times \frac{4}{3}\pi r^3}{a^3} \times 100\%$$

$$= \frac{2 \times \frac{4}{3}\pi \times \left(\frac{\sqrt{3}}{4}\right)^3 a^3}{a^3} \times 100\%$$

$$= 68\%$$

Hence, we can say that 68% volume of the unit cell of Bcc is occupied by atoms and remaining 32% volume is vacant. Thus the Packing Density is 68%. Since the packing density is greater than simple than cubic, it has tightly packed structure, when compared to Sc.

8. **Assuming that atoms in a crystal structure are arranged in close-packet spheres, what is the ratio of the volume of the atoms to the volume available for the face centered cubic structure? Assume a one-atom basis.** ***[TU Microsyllabus 2074, P; 22.5]***

Solution:

For a body centred cubic unit cell, the atomic radius can be calculated from figure 68 as follows.

Consider the triangle ABC,

$$AC^2 = AB^2 + BC^2$$

or, $(4r)^2 = a^2 + a^2$

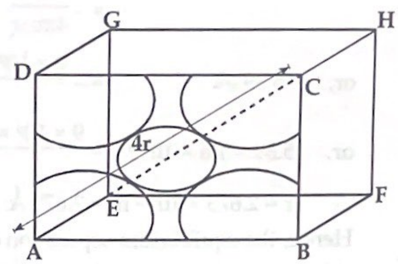
or, $16\, r^2 = 2a^2$

or, $r^2 = \dfrac{2a^2}{16}$

Taking square root on both sides, we have

$$\sqrt{r^2} = \frac{\sqrt{2a^2}}{\sqrt{16}}$$

$\therefore \quad r = \dfrac{a\sqrt{2}}{4}$



[Fig. 68: Representation of face centred cubic structure]

Volume of 4 atoms (v) $= 4 \times \dfrac{4}{3}\pi r^3$

Side of the unit cell (a) $= \dfrac{4r}{\sqrt{2}}$ Since, atomic radius, $r = \dfrac{\sqrt{2}}{4}a$

Volume of the unit cell (V) $= a^3 = \left(\dfrac{4r}{\sqrt{2}}\right)^3$

Packing fraction (P.F.) $= \dfrac{v}{V} \times 100\%$

Substituting for v and V, we have

$$\text{P.F.} = \frac{4 \times \frac{4}{3}\pi r^3}{\left(\frac{4r}{\sqrt{2}}\right)^3} \times 100\%$$

or, $\text{P.F.} = \dfrac{16}{3}\pi r^3 \times \dfrac{2\sqrt{2}}{64 r^3} \times 100\%$

$\therefore \quad \text{P.F.} = \dfrac{\pi\sqrt{2}}{6} = 74\%$

Hence, 74% of the volume of an Fcc unit cell is occupied by atoms and the remaining 26% volume of the unit cell is Vacant. Thus the packing density is 74%. Since the packing density is very high, the Fcc structure has closely (or) tightly packed structure.

9. **The dissociation energy of KF molecule is 15.12 eV. The ionization energy of K is 4.34 eV, and the electron affinity of F is 4.07 eV. What is the equilibrium separation constant for the KF molecule?** ***[TU Microsyllabus 2074, P; 22.9]***

Solution:

Here is given, dissociation energy of KF molecule (D.E.) = 5.12 eV

Ionization energy of K (I.E.) = 434 eV

Electron affinity of F (E.A.) = 4.07 eV

Equilibrium separation constant (r) = ?

Here, to ionize K-atom, energy of 4.34 eV is provided, i.e.,

$$K + 4.34\ \text{eV} \rightarrow K^+ + e^-$$

On the other hand, when F captures one electron, the energy released is, 4.07 eV

$$F + e^- \rightarrow F^- + 4.07\ \text{eV}$$

Here, net energy released (dissociation energy)

= E.A. – Coulomb attraction P.E. – I.E.

5.12 eV = 4.07 eV – Coulomb attraction P.E. – 4.34 eV

Coulomb attraction P.E. = – (5.12 eV + 4.34 eV – 4.07 eV)

= –5.39 eV

$$= -\frac{e^2}{4\pi\epsilon_0 r}$$

$$\text{or,}\quad -5.39\ \text{eV} = -\frac{9\times10^9\times(1.6\times10^{-19})^2}{r}$$

$$\text{or,}\quad 5.39\times1.6\times10^{-19} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{r}$$

$$\therefore\quad r = 2.675\times10^{-10}\ \text{m} = 2.675\ \text{Å}$$

Hence, the equilibrium separation constant for the KF molecule is 2.675 Å.

10. The energy gaps of some alkali halides are KCl = 7.6 eV, KBr = 6.3 eV, KI = 5.6 eV. Which of these are transparent to visible light? At what wave length does each become opaque?

[TU Microsyllabus 2074, P; 24.6]

Solution:

Given, energy gaps of some alkali halides are;

Energy gap of KCl, $(E_g)_{KCl} = 7.6$ eV

Energy gap of KBr, $(E_g)_{KBr} = 6.3$ eV

and energy gap of KI, $(E_g)_{KI} = 5.6$ eV

We know that, the corresponding emission wavelengths are

$$\lambda = \frac{hc}{E_g}$$

$$\lambda_{KCl} = \frac{6.62\times10^{-34}\times3\times10^8}{7.6\times1.6\times10^{-19}}$$

$$= 1.633\times10^{-7}\ \text{m} = 163.3\ \text{nm}$$

Again,

$$\lambda_{KBr} = \frac{6.62\times10^{-34}\times3\times10^8}{6.3\times1.6\times10^{-19}} = 193.0\ \text{nm}$$

$$\text{And, } \lambda_{KI} = \frac{6.62\times10^{-34}\times3\times10^8}{5.6\times1.6\times10^{-19}} = 221.7\ \text{nm}$$

Hence, $\lambda_{visible} > \lambda_{KCl}$, λ_{KBr} and λ_{KI} i.e., E_{gap} of all alkali halides is very much greater than $E_{visible\ light}$ i.e., (2.7 eV to 1.6 eV)

Therefore, all color photons are transmitted through them with no absorption hence they are transparent to visible light.

If λ_{KCl}, λ_{KBr} and λ_{KI} equivalent to (380 – 750) nm they gets absorbed and each become opaque.

Note:

Visible light	
Color	**Wavelength (λ) 'nm'**
V ≈ I	380 – 450
B	450 – 495
G	495 – 570
Y	570 – 590
O	590 – 620
R	620 – 750

11. The density of aluminum is 2.70 gmcm^{-3} and its molecular weight is 26.98 gmole^{-1} (a) Calculate the Fermi Energy (b) If the experimental value of E_F is 12 eV, what is the electron effective mass in aluminum? Aluminum is trivalent. *[TU Microsyllabus 2074, P; 24.8]*

Solution:

Here is given,

Density of aluminum (ρ) = 2.70 gm cm^{-3}

Molecular weight of Aluminum (m) = 26.98 gm mole^{-1}

a. Fermi energy (E_F) = ?

b. Fermi energy ($E_F^{'}$) = 12 eV = $12 \times 1.6 \times 10^{-19}$ J

Effective mass (m_e^{*}) = ?

We know that,

Number of free electrons per unit volume

$$N = \frac{v\rho N_A}{m}$$

Where,

v = valency of atom

N_A = Avogardro's constant

i.e. $N = \frac{3 \times 2.70 \times (6.02 \times 10^{23})}{26.98} = 1.807 \times 10^{23}$ electrons cm^{-3}

$= 1.807 \times 10^{29}$ electrons m^{-3}

Now, we have relation

$$E_F = \frac{\hbar^2}{2m_e}(3N\pi^2)^{2/3}$$

$$= \frac{\left(\frac{6.626 \times 10^{33}}{2\pi}\right)^2}{2 \times 9.1 \times 10^{-31}}(3 \times 1.807 \times 10^{29} \times \pi^2)^{2/3}$$

$$= 1.867 \times 10^{-18}\ \text{J}$$

$$= \frac{1.867 \times 10^{-18}}{1.6 \times 10^{-19}}\ \text{eV}$$

$\therefore E_F = 11.66$ eV

b. Again, for m_e^{*}

$$E_F^{'} = \frac{\hbar^2}{2m_e}(3N\pi^2)^{2/3}$$

$$m_e^{*} = \frac{\hbar^2}{2E_F^{'}}(3N\pi^2)^{2/3}$$

$$= \frac{\left(\frac{6.62 \times 10^{-34}}{2\pi}\right)^2}{2 \times 12 \times 1.6 \times 10^{-19}}(3 \times 1.807 \times 10^{29} \times \pi^2)^{2/3}$$

$m_e^{*} = 8.847 \times 10^{-31}$ kg

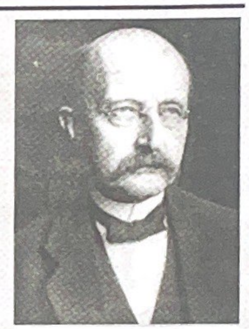
$\therefore m_e^{*} \simeq 0.97\ m_e$

Hence, required Fermi energy and electron effective mass in Aluminum are 11.66 eV and 0.97 m_e respectively.

6

Unit

Semiconductor and Semiconductors Devices



Max Planck
(1858 – 1947)

Learning Objectives:

After studying this chapter, students should be able to:

- Understand the concept of Semiconductor and its types.
- Calculate the carrier concentration and Fermi level of intrinsic and extrinsic Semiconductors.
- Find the electrical conductivity and mobilities of Semiconductors.
- Describe the photo conductivity and metal-metal junction.
- Explain the Semiconductor diode and its band scheme, biasing circuit and V-I characteristics.
- Explain the BJT transistors, its biasing circuits and V-I characteristics.
- Explain the transistor as a voltage amplifier.
- Understand the Field Effect Transistor (FET) and its types.
- Solve the various numerical problems related with Semiconductors.

22. Why is the base of the transistor made thin and slightly doped as compared with emitter and collector regions?

Ans: The thin and slightly doped base gives almost all the free electrons entering from the emitter to base, enough time to diffuse into the collector. Then these electrons flow through the collector to the supply voltage. In most transistors, more than 95% of the emitter electrons flow to be the collector, less than 5% flow out the external base lead. Since the base is lightly doped and very thin, very few electrons manage to recombine and escape into the external base lead giving weak base current. For this reason, the base of the transistor is made thin and lightly doped.

23. Why is the emitter of a transistor doped heavily?

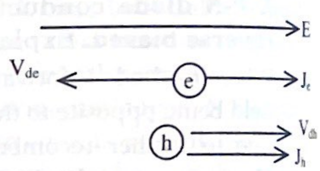
Ans: If the emitter of a transistor is doped heavily, more number of majority charge carriers (holes or electrons) are available in it and they can be easily emitted. The main function of the emitter is to emit the majority charge carriers and pass into the base. It is, possible only if the emitter is heavily doped. Therefore, the emitter of a transistor is heavily doped.

24. What do you mean by biasing a transistor?

Ans: Supplying voltage to a transistor to get it ready for operation is called Biasing of a transistor. A transistor is biased in such as way that it can be used as an amplifier. For this, the emitter base junction must be forwards biased and the collector base junction must be reverse biased. In such biasing, for the same current passing through the emitter and collector of a transistor, the voltage appearing at the collector will be much higher than that appearing at the emitter.

25. Sketch motion of an electron and hole in a solid subject to an electric field.

Ans: In the case of Semiconductor where charge carriers are electrons and holes, both have drift velocity V_{de} and V_{dh} respectively. The velocity of electron V_{de} is in the opposite direction of the electric field where as that of holes V_{dh} is along the direction of the applied field as shown figure 11 below.



[Fig. 46: Sketch the motion of an electron and hole in electric field]

Although the drift velocity of the electrons and holes are opposite, the mobility is defined to be positive for both electrons and holes, i.e.,

$$\mu = \frac{|V_{de}|}{E} \text{ and } \mu_h = \frac{|V_{dh}|}{E}$$

Although the drift velocity of electrons and holes are opposite, their electric current is in the same direction of the applied electric field as shown in figure 46.

Worked Out Examples

1. The energy gap in Silicon is 1.1 eV. The average effective mass is $0.31m_e$. Calculate the electron concentration in the conduction band of Silicon at room temperature 300k. Assume, $E_f = \frac{E_g}{2}$. *[TU Microsyllabus 2074, W; 25.1]*

Solution:

The energy gap in Silicon (E_g) = 1.1 eV

The average effective mass $(m^*) = 0.31m_e$

Electron concentration (n) = ?

We know that, the electron concentration of Silicon Semiconductor

$$n = N_c e^{\frac{E_f - E_g}{K_B T}} \text{ or } N_c e^{\frac{-(E_g - E_f)}{K_B T}}$$

Where, $N_c = 2\left(\frac{m_e K_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$

or, $$n = 2\left(\frac{m^* K_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} e^{\frac{E_f - E_g}{K_B T}}$$

Since, $E_c = E_g$ when $E_v = 0$

$$n = 2\left\{\frac{0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{2\pi \times (1.05 \times 10^{-34})^2}\right\}^{\frac{3}{2}} \times e^{\frac{-0.55 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-34} \times 300}}$$

Where, $m^* = 0.31\ m_e$ and $E_f = \frac{E_g}{2}$.

$\therefore$ $n = 2.6 \times 10^{15}$ per m^3.

Hence, required electron concentration in the conduction band at room temperature is 2.6×10^{15} per m^3.

Since, electron concentration of typical metal $\approx 10^{28}$ electron / m^3.

2. **A sample of Si is dipped with Phosphorous. The donor impurity level lies 0.045 eV below the bottom of the conduction band. At T = 300K, E_f is 0.010 eV above the donor level. Calculate (a) the impurity concentration (b) the number of ionized impurities (c) the free electron concentration and (d) hole concentration (For Si, E_g = 1.100 eV, m_e^* = 0.31 m_e, m_h^* = 0.38m_e)** *[TU Microsyllabus 2074, W; 25.2]*

Solution:

Here, we assumed energy of valence band (E_v) = 0. So, $E_g = E_c$ Since, $E_g = E_c - E_v$.

Then, Fermi energy (E_f) = 1.1 eV – 0.045 eV + 0.010 eV

∴ E_f = 1.065 eV.

a. For N_d calculation, we know that the relation

$$N_c e^{\frac{E_f - E_c}{K_BT}} = N_v e^{\frac{E_v - E_f}{K_BT}} + N_d\left[1 - \frac{1}{e^{\frac{E_d - E_f}{K_BT}} + 1}\right]$$

$$\text{or,}\quad N_c e^{\frac{E_f - E_c}{K_BT}} = N_v e^{\frac{-E_f}{K_BT}} + N_d\left[1 - \frac{1}{e^{\frac{E_d - Ef}{K_BT}} + 1}\right] \quad \dots (1)$$

To find, $N_c = 2\left(\frac{m_e^* K_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}$

$$N_c = 2\left(\frac{2\pi\, m_e^* K_BT}{h^2}\right)^{\frac{3}{2}}$$

$$N_c = 2\left\{\frac{2 \times 3.14 \times 0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.62 \times 10^{-34})^2}\right\}^{\frac{3}{2}}$$

∴ $N_c = 4.39 \times 10^{24}\ m^{-3}$. Again $N_v = 2\left(\frac{m_h^* K_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}$

$$N_v = 2\left(\frac{2\pi\, m_h^* K_BT}{h^2}\right)^{\frac{3}{2}} = 2\left\{\frac{2 \times 3.14 \times 0.38 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.62 \times 10^{-34})^2}\right\}^{3/2}$$

∴ $N_v = 5.95 \times 10^{24}\ m^{-3}$.

Substituting for N_c, N_v, E_f and E_d in equation (1), we get

$$4.39 \times 10^{24}\, e^{\frac{1.065 - 1.100}{0.026}} = 5.95 \times 10^{24}\, e^{\frac{-1.065}{0.026}} + N_d\left[1 - \frac{1}{e^{\frac{-0.010}{0.026}} + 1}\right]$$

Where, $K_BT = 1.38 \times 10^{-23} \times 300 = 0.026$ eV

or, $1.13 \times 10^{24} = 9.5 \times 10^{6} + 0.41\, N_d$

∴ $N_d = 2.79 \times 10^{24}\ m^{-3}$

b. The number of ionized impurities are given by

$$N_d^+ = N_d\left[1 - \frac{1}{e^{\frac{E_d - E_f}{K_BT}} + 1}\right] = 2.7 \times 10^{24}\left[1 - \frac{1}{e^{\frac{-0.010}{0.025}} + 1}\right]$$

∴ $N_d = 1.08 \times 10^{24}\ m^{-3}$

c. The free electron concentration

$$n = N_c e^{\frac{E_f - E_c}{K_BT}} = 4.39 \times 10^{24}\, e^{\frac{(1.065 - 1.100)}{0.026}} = 1.08 \times 10^{24}\ m^{-3}$$

d. The hole concentration $P = N_v e^{\frac{E_v - E_f}{K_BT}} = 5.95 \times 10^{24}\, e^{\frac{-1.065}{0.026}} = 1.88 \times 10^{6}\ m^{-3}$

Hence, required impurity concentration, number of ionized impurities, free electron concentration and hole concentration are $2.79 \times 10^{24} m^{-3}$, $1.08 \times 10^{24}\ m^{-3}$, $1.08 \times 10^{24}\ m^{-3}$ and $1.88 \times 10^{6}\ m^{-3}$ respectively.

3. The band gap in pure germanium is $E_g = 0.67$ eV.

a. Calculate the number of electrons per unit volume in the conduction band at 250 K, 300 K, and at 350 K.

b. Do the same for Silicon assuming $E_g = 1.1$ eV. The effective mass of the electrons in germanium is 0.12 m and in Silicon 0.31 m, where m is the free electron mass.

[TU Microsyllabus 2074, P; 25.1]

Solution:

Here is given, band gap in pure germanium $(E_g)_{Ge} = 0.67$ eV

Fermi energy in pure germanium $(F_F)_{Ge} = \frac{E_g}{2} = \frac{0.67}{2} = 0.33$ eV

a. Number of electrons per unit volume (N_e) = ?

$T_1 = 250$ K

$T_2 = 300$ K

$T_3 = 350$ K

We know that,

$$N_e = N_C e^{-\frac{(E_g - E_F)}{k_B T}}$$

$$= \frac{1}{4}\left(\frac{2m_e^* k_B T}{\hbar^2 \pi}\right)^{\frac{3}{2}} e^{-\frac{(E_g - E_F)}{k_B T}}$$

For germanium

$$m_e^* = 0.12\, m_e$$

$$(N_e)_{Ge} = \frac{1}{4}\left(\frac{2 \times 9.1 \times 10^{-31} \times 0.12 \times 1.38 \times 10^{-23}\, T}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} e^{-\frac{(0.67 - 0.335) \times e}{1.38 \times 10^{-23}\, T}}$$

$$= \frac{1}{4}\left(\frac{2 \times 9.1 \times 10^{-31} \times 0.12 \times 1.38 \times 10^{-23}\, T}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} e^{-\frac{(0.67 - 0.335) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}\, T}}$$

$$= 2.03 \times 10^{20}\, T^{\frac{3}{2}}\, e^{-\frac{3.887 \times 10^3}{T}}$$

Now for $T_1 = 250$ K

$(N_e)_{Ge}$ at 250 K

$$= 2.03 \times 10^{20}\, (250)^{\frac{3}{2}}\, e^{-\frac{3.887 \times 10^3}{250}}$$

$\therefore$ $(N_e)_{Ge} = 1.518 \times 10^{17}$ electrons m^{-3}

Again at 300 K

$$(N_e)_{Ge} = 2.03 \times 10^{20}\, (300)^{\frac{3}{2}}\, e^{-\frac{3.887 \times 10^3}{300}}$$

$(N_e)_{Ge}$ at 300 K $= 2.63 \times 10^{18}$ electrons m^{-3}

And, $(N_e)_{Ge}$ at 350 K

$$(N_e)_{Ge} = 2.03 \times 10^{20}\, (350)^{\frac{3}{2}}\, e^{-\frac{3.887 \times 10^3}{350}}$$

$$= 2.03 \times 10^{20} \times (350)^{\frac{3}{2}}\, e^{-11.057} \qquad \text{Since, } e^{-\frac{3.887 \times 10^3}{350}} = 1.577 \times 10^{-5}$$

$\therefore$ $(N_e)_{Ge} = \simeq 2.097 \times 10^{19}$ electrons m^{-3}

b. For Silicon

$E_g = 1.1$ eV $= 1.1 \times 1.6 \times 10^{-19}$ J

$E_F = 0.55$ eV $= 0.55 \times 1.6 \times 10^{-19}$ J

$m_e^* = 0.31\, m_e$

$(N_e)_{Si}$ at 250 K, 300 K and 350 K = ?

We know that,

$$= \frac{1}{4}\left(\frac{2m_e^* k_B T}{\hbar^2 \pi}\right)^{\frac{3}{2}} e^{-\frac{(E_g - E_F)}{k_B T}}$$

$$= \frac{1}{4}\left(\frac{2 \times 9.1 \times 10^{-31} \times 0.31 \times 1.38 \times 10^{-23}\, T}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} e^{-\frac{(1.1 - 0.55) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}\, T}}$$

$$= 8.445 \times 10^{20}\, T^{\frac{3}{2}}\, e^{-\frac{6.382 \times 10^3}{T}}$$

Now, $(N_e)_{Si}$ at 250 K

$$(N_e)_{Si}\ 8.445 \times 10^{20}\ (250)^{\frac{3}{2}}\ e^{-\frac{6.382 \times 10^3}{250}} = 2.73 \times 10^{13} \text{ electrons m}^{-3} \qquad \left[\because e^{-\frac{6.382 \times 10^3}{250}} = 8.19 \times 10^{-12}\right]$$

Again, $(N_e)_{Si}$ at 300 K

$$(N_e)_{Si}\ 300\text{ K} = 8.445 \times 10^{20}\ (300)^{\frac{3}{2}}\ e^{-\frac{6.382 \times 10^3}{300}} = 2.53 \times 10^{15} \text{ electron m}^{-3}$$

And, $(N_e)_{Si}$ at 350 K

$$(N_e)_{Si} \text{ at } 350\text{ K} = 8.445 \times 10^{20}\ (350)^{\frac{3}{2}}\ e^{-\frac{6.382 \times 10^3}{350}} = 6.66 \times 10^{16} \text{ electrons m}^{-3}.$$

Hence, number of electrons per unit volume in the condition band for germanium and Silicon are obtained as above.

4. **Suppose that the effective mass of hole in a material is four times that of electrons. At what temperature would the Fermi level shifted by 10% from the middle of the forbidden energy gap? Let, E_g = 1 eV.** *[TU Microsyllabus 2074, P; 25.2]*

Solution:

Here, given, effective mass of holes $(m_h^*) = 4 \times$ effective mass of electron (m_e^*)

Temperature (T) = ?

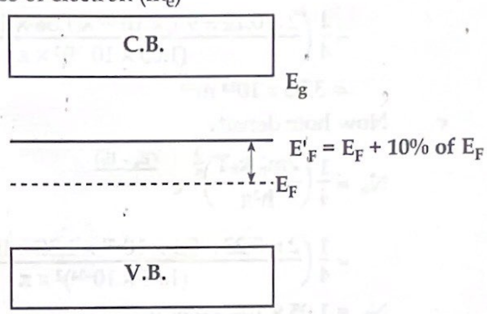
Energy gap (E_g) = 1 eV

According to question,

Fermi level shifted by 10%

$$E'_F = E_F + 10\%\ E_F$$

$$= \frac{E_g}{2} + 10\%\ \frac{E_g}{2}$$



[Fig. 47: Energy band diagram]

We know that,

$$E'_F = \frac{E_g}{2} + \frac{3}{4} k_B T\ \ell n\left(\frac{m_h^*}{m_e^*}\right)$$

or, $E_F + 10\%\ E_F = \frac{E_g}{2} + \frac{3}{4} k_B T\ \ell n\left(\frac{m_h^*}{m_e^*}\right)$

or, $0.5 \times 1.6 \times 10^{-19} + \frac{10}{100} \times 0.5 \times 1.6 \times 10^{-19} = \frac{1 \times 1.6 \times 10^{-19}}{2} + \frac{3k_BT}{4}\ \ell n\left(\frac{4m_e^*}{m_e^*}\right)$

or, $(0.5 + 0.05) \times 1.6 \times 10^{-19} = 0.5 \times 1.6 \times 10^{-19} + \frac{3k_BT}{4}\ \ell n\ (4)$

or, $0.05 \times 1.6 \times 10^{-19} = \frac{3k_BT}{4}\ \ell n\ (4)$

or, $T = \frac{4 \times 0.05 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23} \times \ln 4}$

$= 558.058$

$\therefore$ T = 558.1 K

Hence, at 558.1 K temperature the Fermi level shifted by 10% from the middle of the forbidden energy gap.

5. **The energy gap in germanium is 0.67 eV. The electron and the hole effective masses are 0.12 m and 0.23m respectively, where m is the free electron mass. Calculate (a) the Fermi Energy, (b) the electron density, and (c) the hole density at T = 300 K.** *[TU Microsyllabus 2074, P; 25.3]*

Solution:

Here, given energy gap in germanium $(E_g)_{Ge}$ = 0.67 eV = $0.067 \times 1.6 \times 10^{-19}$ J

Effective mass of electron $(m_e^*) = 0.12\text{ m} = 0.12 \times 9.1 \times 10^{-31}$ kg

Effective mass of hole $(m_h^*) = 0.23\text{ m} = 0.23 \times 9.1 \times 10^{-31}$ kg

a. The Fermi energy (E_F) = ?
b. Electron Density (N_e) = ?
c. Hole density (N_h) = ?

Temperature (T) = 300 K

We know that,

a. $$E_F = \frac{E_g}{2} + \frac{3}{4} k_B T \, \ell n \left(\frac{m_h^*}{m_e^*}\right)$$

$$= \frac{0.67 \times 1.6 \times 10^{-19}}{2} + \frac{3}{4} \times 1.38 \times 10^{-23} \times 200 \, \ell n \left(\frac{0.23 \text{ m}}{0.12 \text{ m}}\right)$$

$$= 5.562 \times 10^{-20} \text{ J}$$

$$= 3.472 \times 10^{-1} \text{ eV}$$

$\therefore \quad E_F \simeq 0.347$ eV

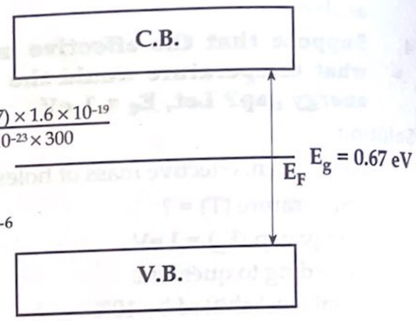
We have a relation,

b. $$N_e = \frac{1}{4}\left(\frac{2m_e^* k_B T}{\hbar^2 \pi}\right)^{\frac{3}{2}} e^{-\frac{(E_g - E_F)}{k_B T}}$$

$$= \frac{1}{4}\left(\frac{2 \times 0.12 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} e^{-\frac{(0.67 - 0.347) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$$

$$= \frac{1}{4}\left(\frac{2 \times 0.12 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} \times 3.478 \times 10^{-6}$$

$$\simeq 3.96 \times 10^{18} \text{ m}^{-3}$$



[Fig. 48: Representation of energy gap and Fermi levels]

c. Now hole density

$$N_h = \frac{1}{4}\left(\frac{2m_h^* k_B T}{\hbar^2 \pi}\right)^{\frac{3}{2}} e^{-\frac{(E_g - E_F)}{k_B T}}$$

$$= \frac{1}{4}\left(\frac{2 \times 0.23 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(1.05 \times 10^{-34})^2 \times \pi}\right)^{\frac{3}{2}} \times 3.478 \times 10^{-6}$$

$\therefore \quad N_h = 1.05 \times 10^{19}$ holes m^{-3}.

Hence, required Fermi energy, electron and hole densities are 0.347 eV, 3.96×10^{18} electrons m^{-3} and 1.05×10^{19} holes m^{-3} respectively.

6. A certain intrinsic Semiconductor has a band gap E_g is 0.2 eV. Measurement shows that it has a resistivity at room temperature 300 K of 0.3 Ωm. What would you predict its resistivity to be at 350 K? ***[TU Microsyllabus 2074, P; 25.13]***

Solution:

Here, given band gap of Semiconductor (E_g) = 0.2 eV = $0.2 \times 1.6 \times 10^{-19}$ J

Initial temperature (T_1) = 300 K

Final temperature (T_2) = 350 K

Initial resistivity (ρ_1) = 0.3 Ω m

Final resistivity (ρ_2) = ?

We know that, for an intrinsic Semiconductor,

$$N_i = N_e = N_h = 2\left(\frac{2\pi k_B T}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} e^{-\frac{E_g}{2k_B T}}$$

Conductivity (σ) = $eN_i(\mu_e + \mu_n)$

Resistivity (ρ) = $\dfrac{1}{eN_i(\mu_e + \mu_n)}$

Then, $$\frac{\rho_1}{\rho_2} = \frac{(N_i)_2 \, e(\mu_e + \mu_h)}{(N_i)_1 \, e(\mu_e + \mu_h)}$$

or, $$\frac{\rho_1}{\rho_2} = \frac{\left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{2\pi k_B T_2}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} e^{-\frac{E_g}{2k_B T_2}}}{\left(\frac{1}{2}\right)^{\frac{1}{2}}\left(\frac{2\pi k_B T_1}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*)^{\frac{3}{4}} e^{-\frac{E_g}{2k_B T_1}}}$$

or, $\dfrac{\rho_1}{\rho_2} = \left(\dfrac{T_2}{T_1}\right)^{\frac{3}{2}} e^{-\frac{E_g(T_1 - T_2)}{2k_BT_1T_2}}$

or, $\dfrac{0.3}{\rho_2} = \left(\dfrac{350}{300}\right)^{\frac{3}{2}} e^{-\frac{0.2 \times 1.6 \times 10^{-19} (300 - 350)}{2 \times 1.38 \times 10^{-23} \times 300 \times 350}}$

or, $\dfrac{0.3}{\rho_2} = 2.189$

or, $\rho_2 = \dfrac{0.3}{2.189}$

$\therefore \quad \rho_2 = 0.136\ \Omega$ m

Hence, required resistivity at 350 K is 0.136 Ω m.

7. **The energy gap in Silicon is 1.1 eV, where as in diamond it is 6 eV. What conclusion can you draw about the transparency of the two materials to visible, light (4000 Å to 7000Å).**
[TU Microsyllabus 2074, P; 25.16]

Solution:

Here, given energy gap in Silicon $(E_g)_{Si} = 1.1 \text{ eV} = 1.1 \times 1.6 \times 10^{-19}$ J

Energy gap in diamond $(E_g)_{dia} = 6 \text{ eV} = 6 \times 1.6 \times 10^{-19}$ J

Wavelength of visible light = 4000 Å to 7000 Å

We know that,

Band gap energy, $(E_g) = \dfrac{hc}{\lambda_c}$

For Silicon, $(E_g)_{Si} = \dfrac{hc}{(\lambda_c)_{Si}}$

or, $(\lambda)_{Si} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.1 \times 1.6 \times 10^{-19}}$

$= 1.1290 \times 10^{-10} = 1129$ nm

$\therefore (\lambda)_{Si} = 11290$ Å

Hence all visible lights are absorbed since $(\lambda_c)_{Si} < \lambda_{visible}$ But it can transmit infrared light having wavelength $= 1.1 \times 10^{-6}$ m.

For diamond, $(\lambda_c)_{dia} = \dfrac{hc}{(E_g)_{dia}}$

$= \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8}{6 \times 1.6 \times 10^{-19}}$

$= 2.066 \times 10^{-7} \text{ m} = 2066 \times 10^{-10}$ m

$\therefore (\lambda_c)_{dia} = 2066 \text{ Å} = 206$ nm

$(\lambda_c)_{dia} < \lambda_{visible}$ i.e. E_{photon} (visible light) $< E_g$

Hence, in the case of diamond all visible lights are transmitted by such type of non metallic materials. Thus, Diamond appears transparent and colourless.

8. **The current through p-n junctions is 1×10^{-8} A when a reverse bias voltage of 10 V is applied across the junction at T = 300 K. What will be the current through the diode when a forward bias voltage of (a) 0.1 V, (b) 0.3 V and (c) 0.5 V is applied?**
[TU Microsyllabus 2074, P; 26.1]

Solution:

Here, given current through p-n junction $(i_r) = 1 \times 10^{-8}$ A

Reverse bias voltage $(V_r) = 10$ V

Temperature (T) = 300 K

Current through p-n junction diode at different voltage

$(i_f)_{0.1V} = ?$

$(i_f)_{0.3V} = ?$

$(i_f)_{0.5V} = ?$

We know that diode equation

$$i = i_0 \left[e^{\frac{|e|V_r}{k_B T}} - 1 \right]$$

Where, i_0 = current associated with the flow of minority carriers (electrons) from the P side to the N side called reverse saturation current.

For reverse biased

$$i_r = i_0 \left[e^{\frac{|e|V_r}{k_B T}} - 1 \right]$$

or, $$1 \times 10^{-8} = i_0 \left[e^{\frac{1.6 \times 10^{-19} \times 10}{k_B T}} - 1 \right]$$

$$= i_0 \left[e^{\frac{1.6 \times 10^{-19} \times 10}{1.38 \times 10^{-23} \times 300}} - 1 \right]$$

$$= i_0 [0 - 1]$$

$\therefore \quad i_0 = -1 \times 10^{-8}$ A.

Negative sign indicates that the net electron flow is from P side nN side.

For forward biased case

Reverse saturation current (i_0) = 1×10^{-8} A

a. Forward voltages $(V_f)_a = 0.1$ V

Then forward current through the diode

$$(i_f)_a = i_0 \left[e^{-\frac{|e|V_a}{k_B T}} - 1 \right]$$

$$= 1 \times 10^{-8} \left[e^{-\frac{1.6 \times 10^{-19} \times 0.1}{1.38 \times 10^{-19} \times 300}} - 1 \right]$$

$$= 1 \times 10^{-8} [47.85 - 1]$$

$$= 4.68 \times 10^{-7} \text{ A}$$

$$= 0.468\ \mu\text{A}$$

b. When $(V_f)_b = 0.3$ V

Forward current $(V_f)_b$ $$= i_0 \left[e^{-\frac{|e|V_b}{k_B T}} - 1 \right]$$

$$= 1 \times 10^{-8} \left[e^{\frac{1.6 \times 10^{-19} \times 0.3}{1.38 \times 10^{-19} \times 300}} - 1 \right]$$

$$= 1.078 \times 10^{-3} \text{ A}$$

$$= 1.08 \text{ mA}$$

c. When $(V_f)_c = 0.5$V

Forward current $(i_f)_c$ $$= i_0 \left[e^{\frac{|e|V_c}{k_B T}} - 1 \right]$$

$$= 1 \times 10^{-8} \left[e^{\frac{1.6 \times 10^{-19} \times 0.5}{1.38 \times 10^{-23} \times 300}} - 1 \right]$$

$$= 2.509 \text{ A}$$

Hence, required current at forward voltage 0.1 V, 0.3 V and 0.5 V are 0.468μA, 1.08 mA and 2.509 A respectively, From above it is concluded that, if the diode is forward biased, the current increase very rapidly with increasing voltage.

9. In the ideal diode the reverse saturation current should be as small as possible. Considering the fact that E_g for Si is 1.1 eV and E_g for Ge is 0.67 eV. Which material is better suited for the fabrication of p-n junction diodes? ***[TU Microsyllabus 2074, P; 26.2]***

Solution:

Here, given band gap of Si $(E_g)_{Si} = 1.1$ eV

Band gap of Ge $(E_g)_{Ge} = 0.67$ eV

We know that diode equation,

$$i = i_0\left(e^{\frac{|e|V_c}{k_BT}} - 1\right)$$

$$= i_0\left(e^{\frac{E_g}{k_BT}} - 1\right)$$

For Si

$$i_{Si} = i_0\left(e^{\frac{1.1 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times T}} - 1\right)$$

Let at normal temperature T = 300 K

$$= 1 \times 10^{-8}\left(e^{\frac{1.1 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}} - 1\right)$$

$$i_{Si} = i_0\,[3.01 \times 10^{11} - 1]$$

$$= 3.01 \times 10^{18}\, i_0$$

Again, for Ge

$$i_{Ge} = i_0\left(e^{\frac{0.67 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}} - 1\right)$$

$$= i_0\,[1.80 \times 10^{11} - 1]$$

$$\simeq 1.80 \times 10^{11}\, i_0$$

Hence, Silicon is better suited for the fabrication of p-n junction diode, Silicon is much better for high current application as it has very high forward current whereas germanium diode have very small forward current.

10. The reverse saturation current of a Silicon diode is $i_0 = 5 \times 10^{-9}$ A. The voltage across the diode when forward biased is 0.45 V. (a) What is the current through the diode at T = 27°C? (b) If the voltage across the diode is held constant, and we assume that i_0 does not change with temperature, what is the current through the diode at T = 47° C?

[TU Microsyllabus 2074, P; 26.3]

Solution:

Here, given reverse saturation current (i_0) = 5×10^{-9} A

Forward biased voltage (V) = 0.45 V

a. Current through diode at 27°C (i_{27}) = ?

Initial temperature (T_1) = 27°C = 300 k

b. Current through diode at 47°C (i_{47}) = ?

We know that diode equation,

$$i = i_0\left[e^{+\frac{|e|V}{k_BT}} - 1\right]$$

$$\text{Then } i_{27} = i_0\left[e^{\frac{1.6 \times 10^{-19} \times 0.45}{1.38 \times 10^{-23} \times 300}} - 1\right]$$

$$= 5 \times 10^{-9}\left[e^{\frac{1.6 \times 10^{-19} \times 0.45}{1.38 \times 10^{-23} \times 300}} - 1\right]$$

$$= 1.81 \times 10^{-1}$$

$$= 0.181 \text{ A}$$

Again,

$$i_{47} = 5 \times 10^{-9}\left[e^{\frac{1.6 \times 10^{-19} \times 0.45}{1.38 \times 10^{-23} \times 320}} - 1\right]$$

$$= 6.11 \times 10^{-2} \text{ A}$$

$$= 0.0611 \text{ A}$$

Hence, required current across the diode at 27°C and 47°C are 0.181 and 0.0611 A respectively.

11. In problem 10, we assumed that the reverse saturation current i_o remains constant when the temperature changes. (a) Show that this assumption is highly incorrect by calculating i_o at T = 47°C when $i_o = 5 \times 10^{-9}$ A at 27°C. Assume that the Fermi level on the p-side of the junction is 1 eV below the bottom of the conduction band? (b) If the voltage across the forward biased diode is 0.45 V, as in problem 26.3, what is the current through the diode at T = 47°C? ***[TU Microsyllabus 2074, P; 26.4]***

Solution:

Here is given,

Initial temperature (T_1) = 27°C = 300 K

Final temperature (T_2) = 47°C = 320 K

Saturation current at T_1, i_o = (T = 300 K) = 5×10^{-9} A

Fermi level on the p-side of the junction lies below the bottom of the conduction band.

(E_1) = 1 eV = $1 \times 1.6 \times 10^{-19}$ J

Voltage across the forward biased diode = 0.45 V

Saturation current at 320 K

i_o (T = 320 K) = ?

Current through the diode at 320 K

i_o (T = 320 K) = ?

a. We know that,

Saturation current

$$(i_o)_{300K} = A\, e^{\frac{E_1}{K_B T_1}}$$

or, $$5 \times 10^{-9} = Ae^{-\frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$$

or, $A = 3.0 \times 10^{8}$ A

Now,

$$i_o\ (T = 45°C) = 3.04 \times 10^{8}\ e^{\frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}} = 5.59 \times 10^{-8}\ A$$

∴ i_o (T = 320 K) ≃ 5.6×10^{-8} A

b. For current through the diode at 320 K

We know that,

$$i = i_o \left[e^{\frac{|e|V}{K_B T}} - 1\right]$$

$$= 5.6 \times 10^{-8} \left[e^{\frac{1.6 \times 10^{-19} \times 0.45}{1.38 \times 10^{-23} \times 320}} - 1\right]$$

∴ i = 0.674 A

Hence, from above it is concluded that saturation current varies with temperature i.e., increases with increasing temperature. The current through the diode at 47°C was found to be 0.674A.

12. The reverse saturation current of a Silicon diode doubles when the temperature changes from 27°C to 33°C. What is the position of the Fermi level on the p-side of the junction? ***[TU Microsyllabus 2074, P; 26.5]***

Solution:

Here is given,

Reverse saturation current at 27°C

$(i_o)_{27}$ = i (say)

Reverse saturation current at 33°C

$(i_o)_{33}$ = 2i

T_1 = 27°C = 300 K

T_2 = 273 + 33 = 306 K

Now, we know that

$i_o = A e^{-\frac{E_1}{K_B T}}$

Again, $(i_o)_{T_1} = Ae^{-\frac{E_1}{K_B T_1}}$

$(i_o)_{T_2} = Ae^{-\frac{E_1}{K_B T_2}}$

Now, $\frac{(i_o)_{T_1}}{(i_o)_{T_2}} = \frac{Ae^{-\frac{E_1}{K_B T_2}}}{Ae^{-\frac{E_1}{K_B T_1}}}$

or, $2 = e^{\frac{E_1}{K_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$

or, $ln\, 2 = \frac{E_1}{K_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

or, $E_1 = \frac{T_1 T_2 \ln 2\, K_B}{T_2 - T_1}$

$= \frac{306 \times 300 \times \ln 2 \times K_B}{306 - 300}$

$\therefore$ $E_1 = 10605.15 \times 1.38 \times 10^{-23}$

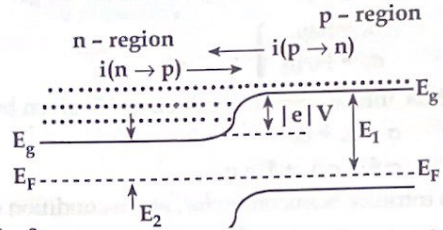
$E_1 = 1.46 \times 10^{-19}$ Joule = 0.91 eV

Here, $E_1 = E_g - E_f$ of the p-side

or, $E_f = E_g - E_1$

or, $E_f = 1.1$ eV – 0.91 eV = 0.2 eV   Since, for Silicon diode $E_g = 1.1$ eV.

Hence, required position of Fermi level 0.2 eV.



[Fig. 49: Representation of different energy levels in N-type and P-type Semiconductor junction]

Additional Numerical Examples

1. Explain how the mobility for electrons and holes are defined? How do you relate mobility with drift velocity for both of the charge carriers in an intrinsic Semiconductor? Show and find an expression to relate conductivity of an intrinsic Semiconductor with the temperature directions?

Solution:

When an electric field E is applied to material, the charge carriers attain a velocity. This velocity is known as drift velocity. The drift velocity is proportional to the applied electric field. i.e.,, $V_d \alpha E$

or, $V_d \alpha \mu E$ ... (1)

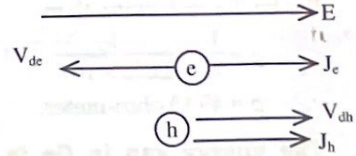
Where, μ is constant known as mobility of charge carriers.

Therefore, $\mu = \frac{|V_d|}{E}$ ... (2)

In the case of Semiconductor where charge carriers are electrons and holes, both have drift velocity V_{de} and V_{dh} respectively. The velocity of electron V_{de} is in the opposite direction of the electric field where as that of holes V_{dh} is along the direction of the applied field as shown below.

Although, the drift velocity of the electrons and holes are opposite, the mobility is defined to be positive for both electrons and holes i.e.,,

$\mu_e = \frac{|V_{de}|}{E}$ and $\mu_h = \frac{|V_{dh}|}{E}$



[Fig. 50: Direction of drift velocity and current density]

Although, the drift velocity of electrons and holes are opposite, their electric current is in the same direction of the applied electric field as shown in figure a below:

The drift current density due to electron

$J_e = N|-e|(V_{de})$

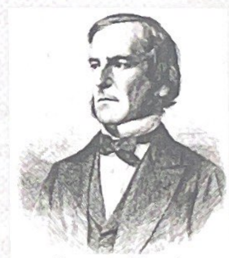
$\therefore$ $J_e = ne\, V_{de}$

$J_e = ne\, \mu_e E$ ... (3)

7

Unit

Universal Gates and Physics of Integrated Circuits



George Boole
(1815 – 1864)

Learning Objectives:

After studying this chapter, students should be able to:

- Understand the concept of Boolean algebra.
- Explain the logic gates and its types.
- Describe the universal gates.
- Explain the TTL, RTL gates.
- Describe the memory and clocks circuits.
- Know about the Semiconductor purification.
- Describe the single crystal growth, process of IC production.
- Understand the electronic component fabrication of a chip.
- Solve the various numerical problems related with universal gates and integrated circuits.

5. **What are logic gates? Give a truth table for a two input NOR gate.**

Ans: A logic gate is a digital circuit which has one or more inputs but only one output. Since, it follows a logical relationship between input and output signals, it is known as a logic gate.

Truth table for NOR gate

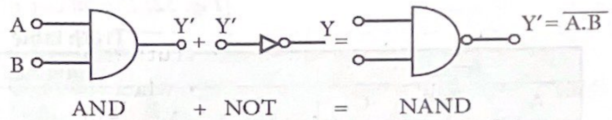
A	B	$Y = \overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

6. **The output of two inputs AND gate is fed to a NOT gate. Give its logic symbol and write down its truth table. Identify the new logic gate formed.**

Ans:

Truth Table

Inputs		Outputs	
A	B	Y' = AB	$Y = \overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0



[Fig. 50: Logic symbol of to inputs NAND gate]

The new logic gate is formed in NAND gate because its truth table is similar liked NAND gate.

Worked Out Examples

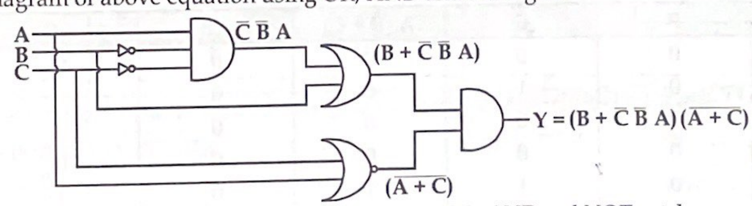
1. **The output of a digital circuit Y is given by the expression Y = $(B + \bar{C}\,\bar{B}\,A)\,(\overline{A + C})$, where A, B and C represent inputs. Draw circuit of above equation using OR, AND and NOT gate. Find its truth table.** *[TU Exam, 2074]*

Solution:

Here, given the output of a digital circuit Y

$$Y = (B + \bar{C}\,\bar{B}\,A)\,(\overline{A + C}),$$ where A, B and C represent inputs.

The circuit diagram of above equation using OR, AND and NOT gate



[Fig. 51: Digital circuit diagram using OR, AND and NOT gate]

Following is the truth table for above circuit.

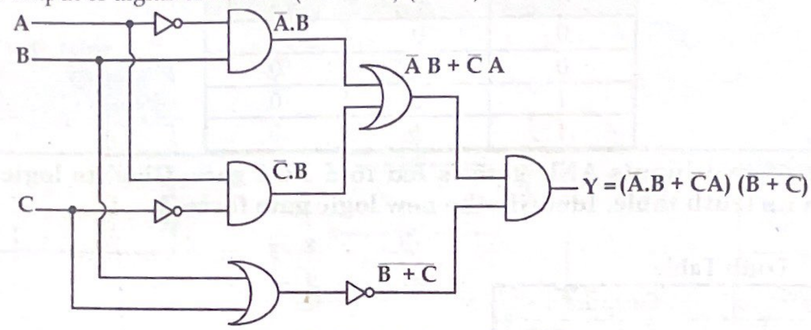
Truth table

Inputs			Intermediate					Output
A	B	C	$\bar{B}$	$\bar{C}$	$\bar{C}\,\bar{B}\,A$	$B + \bar{C}\,\bar{B}\,A$	$\overline{A + C}$	$(B + \bar{C}\,\bar{B}\,A)\,(\overline{A + C})$
0	0	0	1	1	0	0	1	0
0	0	1	1	0	0	0	0	0
0	1	0	0	1	0	1	1	1
0	1	1	0	0	0	1	0	0
1	0	0	1	1	1	1	0	0
1	0	1	1	0	0	0	0	0
1	1	0	0	1	0	1	0	0
1	1	1	0	0	0	1	0	0

2. The output of a digital circuit Y_1 is given by this expression: $Y = (\bar{A}B + \bar{C}A)\,\overline{(B + C)}$, where A, B and C represent inputs. Draw a circuit of above equation using OR, AND and NOT gate and hence find its truth table. *[TU Model 2074]*

Solution:

Here, given output of digital circuit $Y = (\bar{A}B + \bar{C}A)\,\overline{(B + C)}$



[Fig. 52: Digital circuit diagram]

Truth table

Inputs			Intermediate							Output
A	B	C	B+C	$\overline{B+C}$	$\bar{A}$	$\bar{C}$	$\bar{A}B$	$\bar{C}A$	$Y = (\bar{A}B + \bar{C}A)\,\overline{(B + C)}$
0	0	0	0	1	1	1	0	0	0
0	0	1	1	0	1	0	0	0	0
0	1	0	1	0	1	1	1	0	0
0	1	1	1	0	1	0	1	0	0
1	0	0	0	1	0	1	0	1	1
1	1	0	1	0	0	1	0	1	0
1	1	1	1	0	0	0	0	0	0

3. Make the appropriate truth tables to prove the following distributive law of Boolean algebra; A (B + C) = AB + AC *[TU Microsyllabus 2074, P; 27.1]*

Solution:

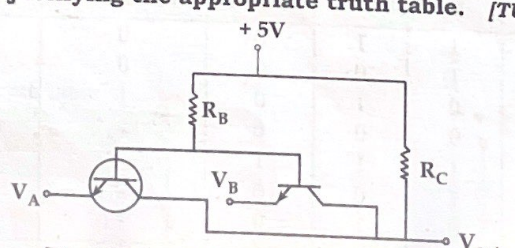
The given Boolean algebra is A (B + C) = AB + AC.

Here, we want to prove the distribution law of Boolean algebra by using truth table;

A	B	C	A (B + C)	AB	AC	AB + AC
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
1	0	0	0	0	0	0
1	0	1	1	0	1	1
1	1	0	1	1	0	1
1	1	1	1	1	1	1

Hence, given Boolean algebra was verified.

4. Analyze the circuit shown in figure 53. Determine the logic function performed by the circuit by making and justifying the appropriate truth table. *[TU Microsyllabus 2074, P; 27.6]*



[Fig. 53: Circuit diagram of logic function]

Solution:

The logic function performed by the given circuit:

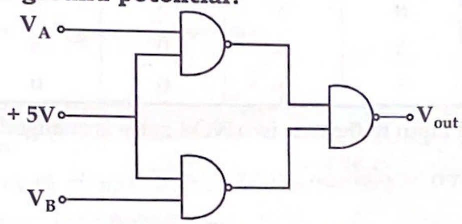
Output $Y = V_A \cdot V_B$ i.e., AND operation

Truth table;

Inputs		Output
$V_A(V)$	$V_B(V)$	$Y = V_A \cdot V_B$ (V)
0	0	0
0	5	0
5	0	0
5	5	5

5. (a) **Find the truth table for the circuit shown in figure 54. What logic function does the circuit perform?**

(b) **What logic function will the circuit perform if the constant + 5V input to the first two gates are changed to ground potential?** *[TU Microsyllabus 2074, P; 27.9]*



[Fig. 54: Given circuit]

Solution:

a. Truth table for the given circuit of logic function:

$Y = \overline{\left[(\overline{5V \cdot V_A}) \cdot (\overline{5V \cdot V_B})\right]}$

$= V_A + V_B$

= OR - operation

Truth table

Inputs (V)			Intermediate (V)				Output (V)
V_A	V_B	+ 5V	$5V \cdot V_A$	$5V \cdot V_B$	$\overline{5V \cdot V_A}$	$\overline{5V \cdot V_B}$	$\overline{(\overline{5V \cdot V_A}) \cdot (\overline{5V \cdot V_B})}$
0	0	+ 5	0	0	+ 5	0	0
0	5	+ 5	0	5	+ 5	5	5
5	0	+ 5	5	0	0	0	5
5	5	+ 5	5	5	0	5	5

b. If the constant + 5V input to the first two gates is changed to ground potential i.e., (+ 0V) then,

Output $Y = \overline{(\overline{5V \cdot V_A}) \cdot (\overline{5V \cdot V_B})}$

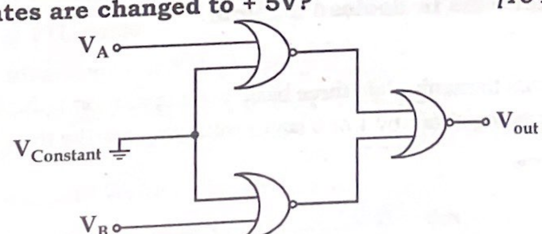
$= \overline{5V \cdot 5V}$

$Y = 0V$

Hence, none of the logic function performed by circuit.

6. (a) **Find the truth table for the circuit of given figure 55. What logic function does the circuit perform?**

(b) **What logic function will the circuit perform if the common grounded input to the first two NOR gates are changed to + 5V?** *[TU Microsyllabus 2074, P; 27.10]*



[Fig. 55: Given circuit]

Solution:

Logic function of given circuit is

$V_{Out} = \overline{(\overline{V_A + 0}) + (\overline{V_B + 0})}$

$= \overline{(\overline{V_A} + \overline{V_B})}$

$= V_A \,.\, V_B$

= AND operation

Truth Table

Inputs (V)			Intermediate (V)				Output (V)
V_A	V_B	$V_{constant}$	$V_A + 0$	$V_B + 0$	$(\overline{V_A + 0})$	$(\overline{V_B + 0})$	$V_{out} = \overline{((\overline{V_A + 0}) + (\overline{V_B + 0}))}$
0	0	0	0	0	5	5	0
0	5	0	0	5	5	0	0
5	0	0	5	0	0	5	0
5	5	0	5	5	0	0	5

b. If the common grounded input to the first two NOR gates is changed to + 5V then

$Y = \overline{((\overline{V_A + 5V}) + (\overline{V_B + 5V}))}$

$= \overline{(\overline{5V} + \overline{5V})}$

$= \overline{0V + 0V}$

= 5 V (High)

It gives no physical meaning in the operation of the circuit.

Additional Numerical Examples

1. Verify the Boolean algebra by truth table method.

a. $\overline{A + B} = \overline{A} \cdot \overline{B}$ **b.** $\overline{A \cdot B} = \overline{A} + \overline{B}$

Solution:

Let A and B are any two inputs in logic circuit. To verify these above Boolean algebra by taking these inputs are either 1 or 0 states as given below:

A	B	$\overline{A}$	$\overline{B}$	A + B	$\overline{A + B}$	A · B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	0	1	0	1	1	1
0	1	1	0	1	0	0	1	1	0
1	0	0	1	1	0	0	1	1	0
1	1	0	0	1	0	1	0	0	0

From above table (a) $\overline{A + B} = \overline{A} \cdot \overline{B}$ and (b) $\overline{A \cdot B} = \overline{A} + \overline{B}$ are verified.

2. Describe the various theorems in Boolean algebra.

Solution:

The Boolean algebra allows us to manipulate three basic logic operation (AND, OR and NOT). Let A, B and C are three variables representing either by 1 or 0 states following are the theorems of Boolean algebra.

i. **Commutation theorems:**

A + B = B + A

AB = BA